Unit 5: Understand the Connections between Proportional Relationships, Lines, and Linear Equations

Cluster: Understand the connections between proportional relationships, lines, and linear equations.

Nevada Academic Content Standard	What does this standard mean that a student will know and be able to do? (adapted from North Carolina *8th Grade Standards, Unpacked Content*)
8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	**8.EE.5** Students build on their work with unit rates from 6th grade and proportional relationships in 7th grade to compare graphs, tables and equations of proportional relationships. Students identify the unit rate (or slope) in graphs, tables and equations to compare two proportional relationships represented in different ways. *Example 1:* Compare the scenarios to determine which represents a greater speed. Explain your choice including a written description of each scenario. Be sure to include the unit rates in your explanation. **Scenario 1:** **Scenario 2:** $y = 55x$ x is time in hours y is distance in miles *Solution:* Scenario 1 has the greater speed since the unit rate is 60 miles per hour. The graph shows this rate since 60 is the distance traveled in one hour. Scenario 2 has a unit rate of 55 miles per hour shown as the coefficient in the equation. Given an equation of a proportional relationship, students draw a graph of the relationship. Students recognize that the unit rate is the coefficient of x and that this value is also the slope of the line.
8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane (introduction to this concept);	**8.EE.6** Triangles are similar when there is a constant rate of proportionality between them. Using a graph, students construct triangles between two points on a line and compare the sides to understand that the slope (ratio of rise to run) is the same between any two points on a line. *Example 1:* The triangle between A and B has a vertical height of 2 and a horizontal length of 3. The triangle between B and C has a vertical height of 4 and a horizontal length of 6. The simplified ratio of the vertical height to the horizontal length of both triangles is 2 to 3, which also represents a slope of $\frac{2}{3}$ for the line, indicating that the triangles are similar.
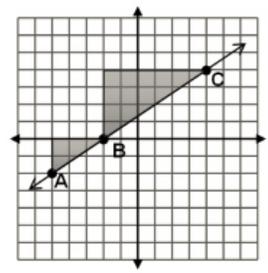
| derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. | Given an equation in slope-intercept form, students graph the line represented.

Students write equations in the form $y = mx$ for lines going through the origin, recognizing that m represents the slope of the line.

Example 2:
Write an equation to represent the graph to the right.

Solution: $y = -\dfrac{3}{2}x$
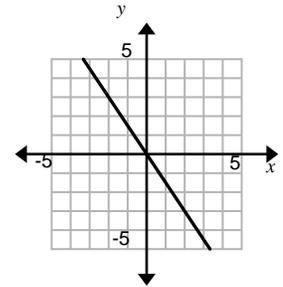
Students write equations in the form $y = mx + b$ for lines not passing through the origin, recognizing that m represents the slope and b represents the y-intercept.

Solution: $y = \dfrac{2}{3}x - 2$
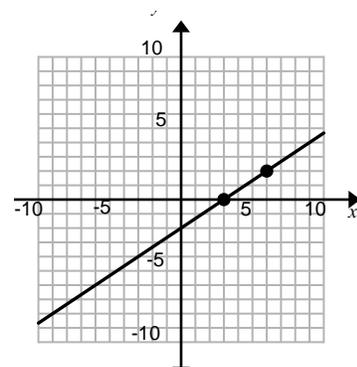
8.F.2 Students compare two functions from different representations.

Example 1:
Compare the following functions to determine which has the greater rate of change. |

Function 1: $y = 2x + 4$ Function 2:

x	y
-1	-6
0	-3
2	3

Solution: The rate of change for Function 1 is 2; the rate of change for Function 2 is 3. Function 2 has the greater rate of change.

8.F.A.2
  Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For*

example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.	*Example 2:* Compare the two linear functions listed and determine which has a negative slope.

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Compare the two linear functions listed and determine which has a negative slope.

Function 1: Gift Card
Samantha starts with $20 on a gift card for the bookstore. She spends $3.50 per week to buy a magazine. Let *y* be the amount remaining as a function of the number of weeks, *x*.

x	*y*
0	20
1	16.50
2	13.00

Function 2: Calculator rental
The school bookstore rents graphing calculators for $5 per month. It also collects a non-refundable fee of $10.00 for the school year. Write the rule for the total cost (*c*) of renting a calculator as a function of the number of months (*m*).
$c = 10 + 5m$

Solution: Function 1 is an example of a function whose graph has a negative slope. Both functions have a positive starting amount; however, in function 1, the amount decreases 3.50 each week, while in function 2, the amount increases 5.00 each month.

NOTE: Functions could be expressed in standard form. However, the intent is not to change from standard form to slope-intercept form but to use the standard form to generate ordered pairs. Substituting a zero (0) for *x* and *y* will generate two ordered pairs. From these ordered pairs, the slope could be determined.

Example 3:
$2x + 3y = 6$

Let $x = 0$: $2(0) + 3y = 6$ Let $y = 0$: $2x + 3(0) = 6$

$$3y = 6$$
$$\frac{3y}{3} = \frac{6}{3}$$
$$y = 2$$
Ordered pair: (0, 2)

$$2x = 6$$
$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$
Ordered pair: (3, 0)

Using (0, 2) and (3, 0) students could find the slope and make comparisons with another function.

8.F.A.3 (part)
Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; ~~give examples of functions that are not linear.~~

8.F.3 Students understand that linear functions have a constant rate of change between any two points. Students use equations, graphs and tables to categorize functions as linear or non-linear.

Example 1:
Determine if the functions listed below are linear or non-linear. Explain your reasoning.
1. $y = -2x^2 + 3$
2. $y = 0.25 + 0.5(x - 2)$

	3.		4.

x	y
1	12
2	7
3	4
4	3
5	4
6	7
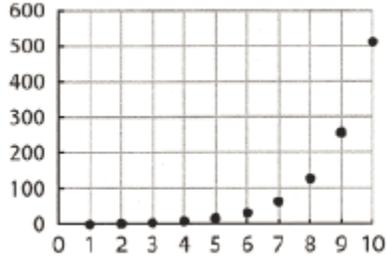
Solution:
1. Non-linear; there is not a constant rate of change
2. Linear; there is a constant rate of change
3. Non-linear; there is not a constant rate of change
4. Non-linear; the graph curves indicating the rate of change is not constant.

Approximate Time Frame: *3 – 4 weeks*

Terms:

- constant of proportionality
- dependent variable
- direct variation
- independent variable
- intercept

- linear function
- proportional relationship
- rate of change
- rise/run
- similar triangles
- slope

- slope-intercept form
- unit rate
- $y = mx$
- $y = mx + b$

Resources

MGH – McGraw Hill, Glencoe Math (2015)
ML – McDougal Littell, Pre-Algebra Book; Larson, 2005
EX – Explorations in Core Math (Holt McDougal)
IL – Illinois *Model Math Curriculum*

NY – *Engage* New York
MAP – Math Assessment Project (MARS)
MA – Massachusetts *Model Curriculum Units - you will have to sign-up for 30 day permission*

	Suggested Topics for Lessons	*Possible Resources*
Prep for unit	**Constant Rate of Change** (proportional and non-proportional) **Slope**	- MGH 3-1 Constant Rate of Change (page 171) - MGH 3-1 Inquiry Lab: *Graphing Technology – Rate of Change* (page 179) - MGH 3-2 *Slope* (page 181) - EX 8-1 *Graphing Linear Equations: Rates of Change* (page 295) - Pearson Video: *Finding Rate of Change Using a Table* - Learn Zillion Lesson: *Determining the constant rate of change* - Pearson Video: *Finding the Rate of Change Using a Graph*

		- Khan Academy: *Slope and Rate of Change* - EX 8-2 *Slope of a Line* (page 299) - ML 8.4 *The Slope of a Line* (page 404) - Khan Academy: *What does the slope represent?* - YouTube Video Song: *Slope of a Line* - YouTube Video Song: *Slope Music Rap*
8.EE.B.6	**Use similar triangles to explain why the slope *m* is the same between any two distinct points on a line** SBAC Evidence: The student uses similar triangles to determine that the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane.	- MGH 3.4 Inquiry Lab *Slope Triangles* (page 207) - Learn Zillion Video Lesson: *Make Lines from Right Triangles* - Khan Academy Video Lesson: *Slope and Triangle Similarity 2* - Slide Share Lesson: *Similar Triangles and Slope* - NYCDOE Series of tasks: *Slippery Slopes* - PBS Learning Media: *Understanding Slope with Similar Triangles* - NY Module 4, Lesson 16: *The Computation of the Slope of a Non-Vertical Line* - MA Lesson 4: *Analyzing Rates of Change Visually and Numerically*
8.EE.B.5 **8.EE.B.6**	**Proportional Relationships** $y = mx$ **Interpreting Unit Rate as Slope** SBAC Evidence: - The student graphs proportional relationships. - The student interprets the unit rate as the slope of the graph of a proportional relationship. - The student finds the equation $y = mx$ for a line.	- MGH 3-3 *Equations in $y = mx$ Form* (page 189) - Learn Zillion *Lesson* Plan: *Graphing Proportional Relationships* - Learn Zillion *Video* Lesson: *Display all possibilities in proportional relationship* - Khan Academy Video & Problems: *Graphing and analyzing proportional relationships* - ML 8.6 *Direct Variation* (page 423) - EX 8-5 *Direct Variation* (page 317) - Learn Zillion *Lesson* Plan: *Interpret Unit Rate as Slope* - Learn Zillion *Video* Lesson: *Find a Unit Rate Using a Graph* - Learn Zillion *Video* Lesson: *Find a Fractional Unit Rate by using a Graph* - Learn Zillion Video Lesson: *Derive $y = mx$ Using Similar Triangles* - Illustrative Math Problems: *Click here* 8.EE.5 Lesson Strickler—Look at Student Crazy Rates: *Click here*

8.EE.B.5	**Compare proportional relationships in different formats** SBAC Evidence: ➢ The student compares two different proportional relationships represented in different formats.	➢ MGH 4-5 *Compare Properties of Functions* (page 309) ➢ YouTube Lesson: *Compare Rates (Slopes) in Different Forms* ➢ Learn Zillion Video Lessons: *Understand Proportional Relationships by Relating Graphs and Equations* ➢ Learn Zillion *Lesson* Plan: *Compare Proportional Relationships*
8.EE.B.6 **8.F.A.3**	**Proportional relationships vs non-proportional relationships** $y = mx + b$ **Slope-Intercept Form** SBAC Evidence: ➢ The student finds/derives the equation $y = mx + b$ for a line. ➢ The student interprets the equation $y = mx + b$ as defining a linear function with a graph that is a straight line	➢ MGH 3-4 *Slope-Intercept Form* (page 199) ➢ MGH 3-5 *Graphing a Line Using Intercepts* (page 209) ➢ ML 8.5 *Slope-Intercept Form* (page 412) ➢ EX 8-3 *Using Slopes and Intercepts* (page 305) ➢ Learn Zillion Video Lessons: *Interpret the Equation y = mx + b* ➢ Learn Zillion Video Lessons: *Interpret the Equation y = mx + b as defining a linear Function* ➢ Illustrative Math Task: *Introduction to Linear Functions* ➢ Video Notes: *Proportional vs Non-proportional Relationships* ➢ YouTube: Introducing Desmos Online Graphing Calculator ➢ Learn Zillion Video Lesson: *Derive y = mx + b Using Similar Triangles* ➢ MAP: *Lines, Slopes and Linear Equations* ➢ YouTube Video Song: *y = mx + b math remix*
8.F.A.2	**Compare functions represented four ways: equation, graph, table, description** SBAC Evidence: ➢ The student recognizes the same function written in different functional forms (algebraic, graphic, tabular, or verbal). ➢ The student compares properties of two functions represented in a different way (algebraic, graphic, tabular, or verbal).	➢ EX 9-4 *Comparing Multiple Representations* (page 361) ➢ IL Unit 4, Lesson 4: *Comparing Functions* ➢ NY Module 5, Lesson 7: *Comparing Linear Functions and Graphs* ➢ Tufts Lesson/Activity: *Who Shares my Function—Linear with All Representations* ➢ Learn Zillion Video Lesson: *Compare two functions by analyzing an equation and a graph* ➢ Learn Zillion Video Lesson: *Compare two functions by analyzing an equation and a table* ➢ Learn Zillion Video Lesson: *Compare two functions by analyzing an equation and a verbal description* ➢ Illustrative Math: *8.F.2 Battery Charging*