OBJECTIVES:

F.IF.A.1 Understand the concept of a function and use function notation.
Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.B.4 Interpret functions that arise in applications in terms of the context.
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

NOTE: This unit can be used as needed (review or introductory) identify and work with functions.

BIG IDEA:
Relationships among quantities can be represented using tables, graphs, verbal descriptions, equations and inequalities. Symbols are used to represent unknowns and variables. We can interpret and make critical predictions from functional relationships.

PREREQUISITE SKILLS:
- students should understand how to evaluate variable expressions
- students should understand how to solve equations with one or two variables

VOCABULARY:
- relation: a set of ordered pairs
- function: a special relation that has a rule that establishes a mathematical relationship between two quantities, called the input and the output. For each input, there is exactly one output
- domain: the collection of all input values
- range: the collection of all output values
- independent variable: the variable in a function with a value that is subject to choice
- dependent variable: the variable in a relation with a value that depends on the value of the independent variable (input)
- function notation: a way to name a function that is defined by an equation. In function notation, the y in the equation is replaced with $f(x)$

SKILLS:

- determine if a given relation is a function
- describe and model functions using an input-output table, mapping diagram, and writing a function rule with and without technology
- determine and differentiate between the domain and range of functions
- use equations of functions to make predictions or interpretations
- evaluate functions using function notation for given values of the variable
- translate among verbal descriptions, graphic, tabular, and algebraic representations of a function with and without technology

REVIEW AND EXAMPLES:

Relation: a set of ordered pairs

Domain: the set of input values (x) in a relation; x is also called the "<u>independent</u>" variable.

Range: the set of output values (y) in a relation; y is also called the "<u>dependent</u>" variable.

Ex 1. State the domain and range using the relation: $\{(-1,2),(0,4),(0,-3),(1,-3)\}$.

To list the domain and range you list them from least to greatest in set notation.

Solution: Domain $\{-1,0,1\}$; Range $\{-3,2,4\}$

Ex 2. Use the following: Popcorn Prices: Small 3.^{00} Medium 4.^{00} Large 5.^{00}

How much would ten large popcorns cost?

$(1, 5.^{00})$, $(2, 10.^{00})$, $(3, 15.^{00})$, ... $(10, ?)$

The total cost depends on the number of popcorns you purchase, so the number of popcorns is the independent variable (input) and the cost is the dependent variable (output). We can write a rule for that to find the cost of any number of large popcorns purchased:

	Cost	equals	$5 (per popcorn)	multiply by	# of popcorns purchased
	C	$=$	$5	$\times$	n
$(10, ?)$	C	$=$	$5	$\times$	10 = $50

Three people in front of you in the line all buy some large popcorn: $(2, 10.^{00})$, $(3, 15.^{00})$ and $(1, 5.^{00})$. You order 3 large popcorns and the popcorn guy says, "That will be $18.^{00}." Is everything functioning here? No, the rule was not followed for your order. The input of 3 large popcorns should have exactly 1 output, $15.

Function: a special type of relation in which each input has exactly one output.
Functions can be represented in several different ways; ordered pairs, table of values, mapping diagrams, graphs and in function notation.

Ordered Pairs: given a relation, it is a function if each input is paired with exactly 1 output (check to see if x repeats).

Ex 3. Is the relation a function? If so, state the domain and range.

a. $\{(3,5),(-4,6),(-2,4),(3,2)\}$

No, the input 3 has 2 output values.

b. $\{(-2,6),(0,10),(1,12),(3,16)\}$

Yes, each input has exactly 1 output.
Domain: {-2, 0, 1, 3}
Range: {6, 10, 12, 16}

Table of Values: given a table of values of a relation, it is a function if each input is paired with exactly 1 output (check to see if x repeats).

Ex 4. Use the following input-output table. Is the relation a function? If so, state the domain and range.

a.

Input	Output
3	0
6	4
9	0
12	−4

Yes, each input has exactly 1 output.
Domain: {3, 6, 9, 12}; Range: {-4, 0, 4}

b.

| Input	Output
2	2
8	6
2	1
10	-6

No, the input 2 has 2 output values.

Mapping Diagrams: given a mapping diagram of a relation, we can tell if it is a function if each input is paired with exactly 1 output.

Ex 5. Is the relation a function? If so, state the domain & range.

a)
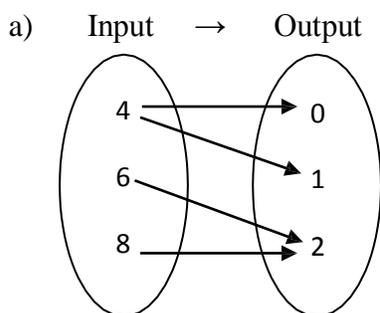
No, the input 4 has two output values.

b)
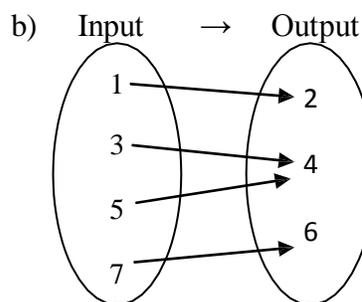
Yes, every input has exactly one output.
Domain: $\{1,3,5,7\}$; Range: $\{2,4,6\}$

Graphs of Functions: Given the graph, we can use the "vertical line test" to determine if a relation is a function.

Vertical Line Test: a graph is a function if all vertical lines intersect the graph no more than once. If you can draw a vertical line between any two points on the graph, then it flunks the vertical line test. The two points would have the same x value, but different y values; which means that there is more than one output (y) for that particular input (x).

Ex 6. Which of the graphs is a function?
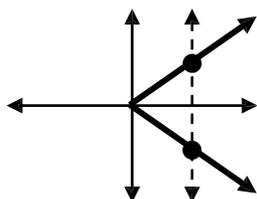
Not a function – fails the vertical line test   Function – passes the vertical test
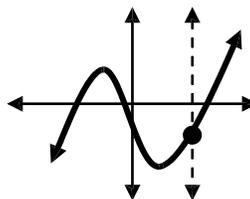
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Function Rule: A function can be represented by an equation that describes the mathematical relationship that exists between the independent (x) and dependent (y) variables.

Ex 7. The equation $y = 2x + 1$, tells us that the output value is equal to 1 more than twice the input value.

We can use the function rule to pair x values with y values and create ordered pairs. Let's input the value 3 into the function rule for x and determine what output (y) value it is paired with.

$$y = 2x + 1$$
$$y = 2(3) + 1$$
$$y = 6 + 1$$
$$y = 7$$

So, the ordered pair (3, 7) is a solution to the function rule.

Function Notation: function notation is a way to name a function that is defined by an equation. For an equation in x and y, the symbol $f(x)$ replaces y and is read as "the value of the function at x" or simply "f of x". Remember, that the x value is the independent variable and the y value is dependent on what the value of x is. So, y is a function of x . $y = f(x)$ or in other words y and the function of x are interchangeable.

Ex 8. Let's look at the equation $y = 2x + 1$. To write it using function notation we replace the y with $f(x)$ since they are interchangeable.   So, the equation $y = 2x + 1$ becomes $f(x) = 2x + 1$.

Why use function notation? It helps us to relate the function rule to its graph. Each solution (ordered pair) to the function rule represents a point that falls on the graph of the function. When using function notation we can see the ordered pair.

Let's use the function rule expressed in function notation to find the value of the function when the input (*x)* is 3.

$$f(x) = 2x + 1$$
$$f(3) = 2(3) + 1$$
$$f(3) = 6 + 1$$
$$f(3) = 7$$

Notice that throughout the process you can see what the input value is. In the final result, you can see the ordered pair. Following the function rule; when *x* has a value of 3, *y* has a value of 7.

Does function notation always have to be expressed as *f(x)*? No. You can use any letter to represent a function. For example; *g(x)*, *h(x)* or *k(x)*. When comparing multiple functions or their graphs you need some way to distinguish between them.

Ex 9. $f(x) = -2x + 3$ $g(x) = x - 4$ $h(x) = 5$ $k(x) = x^2 + 1$

   Evaluate the following expressions given the function rules above.

$g(6)$	$f(-2)$	$h(13)$	$k(0)$
$g(x) = x - 4$	$f(x) = -2x + 3$	$h(x) = 5$	$k(x) = x^2 + 1$
$g(6) = (6) - 4$	$f(-2) = -2(-2) + 3$	$h(13) = 5$	$k(0) = (0)^2 + 1$
$g(6) = 2$	$f(-2) = 4 + 3$		$k(0) = 0 + 1$
	$f(-2) = 7$		$k(0) = 1$

$f(x) - h(x)$	$h(x) \cdot g(x)$	$k(h(x))$	$f(g(x))$
$(-2x + 3) - (5)$	$(5) \cdot (x - 4)$	$k(5)$	$f(x - 4)$
$-2x - 2$	$5x - 20$	$(5)^2 + 1$	$-2(x - 4) + 3$
		$25 + 1$	$-2x + 8 + 3$
		26	$-2x + 11$

ASSESSMENT ITEMS:

1. Explain using the definition of a function why the vertical line tests determine whether a graph is a function.

2. Which of the following tables represent functions?

I.

Input	Output
0	1
0	2
2	3
3	4

III.

Input	Output
−3	2
−2	−2
2	−3
3	3

II.

Input	Output
0	0
1	0
2	1
3	1

IV.

Input	Output
2	−11
0	8
2	4
4	7

A. I and IV only
B. II and III only
C. I, II and III only
D. II, III and IV only

3. Determine which of the following are functions:

I.
II.
III.
IV.
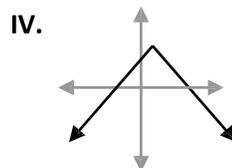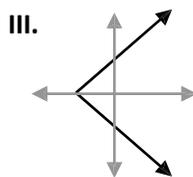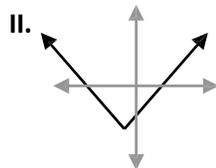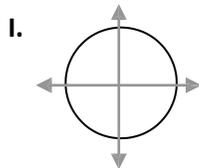
A. I and III only
B. II and IV only
C. II, III, and IV only
D. I, II, III and IV

4. What is the range of the following relation? $\{(3,5),(-2,8),(5,1),(-3,-1)\}$

   A. $\{-1, 1, 5, 8\}$

   B. $\{-3, -2, 3, 5\}$

   C. $\{-1 \le x \le 8\}$

   D. $\{-3 \le x \le 5\}$

5. Use the diagram below when $f(x) = 5$ and $g(x) = 2x + 3.$

   $f(x)$

   $g(x)$
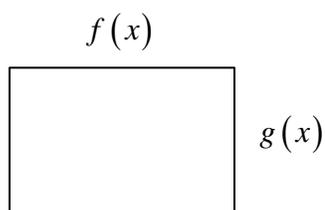
   a. Write algebraic expressions for the area and the perimeter.

   b. If the perimeter is 24 inches, what is the value of x?

6. Compare and contrast a relation and a function.

7. Which input-output table represents the function $f(x) = 2x - 3$?

A.

Input	Output
2	1
3	2
6	9
8	13

B.

Input	Output
2	1
3	2
6	5
8	13

C.

Input	Output
2	1
3	3
6	5
8	13

D.

Input	Output
2	1
3	3
6	9
8	13

8. Let a function be defined as $f(x) = -4x^2 + x - 3$. What is $f(1)$?
   A. -18
   B. -6
   C. 0
   D. 14

9. Kathy has two sets of numbers, *A* and *B*. The sets are defined as follows:

   $A = \{1, 2, 3\}$ $B = \{10, 20, 30\}$

   Kathy created four relations using elements from Set *A* for the domains and elements from Set *B* for the ranges. Which of Kathy's relations is NOT a function?
   A. $\{(1, 10), (1, 20), (1, 30)\}$
   B. $\{(1, 10), (2, 10), (3, 10)\}$
   C. $\{(1, 10), (2, 20), (3, 30)\}$
   D. $\{(1, 10), (2, 30), (3, 20)\}$

10. Translate the following statements into coordinate points.
    $f(-3) = 1$ $g(2) = 4$ $g(0) = 7$ $k(5) = 8$