RATIOS AND PROPORTIONS NOTES

Ratios

A ratio is a comparison of two quantities that have the *same units*. You can express a ratio in any one of the following ways:

$$\frac{18}{5} \qquad\qquad 18:5 \qquad\qquad 18 \text{ to } 5$$

Example #1: If one store has 360 items and another store has 100 of the same items, express the ratio of the items.

$$\frac{360}{100} \qquad \textbf{or} \qquad 360:100 \qquad \textbf{or} \qquad 360 \text{ to } 100$$

Ratios are usually written in lowest terms; therefore, the above example would reduce in this way:

$$\frac{360}{100} \quad \begin{array}{l} \div\ 20 \\ \div\ 20 \end{array} \qquad \text{(What is the largest number you can divide both values by?)}$$

$$\frac{18}{5}$$

Example #2: John earns \$350 a week. His take-home pay, however, is \$295. What is the ratio of his gross pay to his take-home pay.

$$\frac{350}{295} \quad = \quad \frac{70}{59}$$

Rates

A rate is a comparison of two quantities that have *different units*. Rates are usually expressed in the fractional form.

Example: Francine paid \$16 for her 12-month subscription to *Better Homes and Gardens* magazine. Express as a rate.

$$\frac{\$16.00}{12 \text{ magazines}} \quad = \quad \frac{\$4.00}{3 \text{ magazines}}$$

If Francine wants to know how much she pays for each (1) magazine, she can divide \$4 by 3 magazines. This will give her the price per magazine (also called the **unit rate**).

$$\frac{\$4.00}{3} \quad = \quad \$1.33/\text{magazine}$$

Proportions

A proportion is a statement that two ratios or rates are equal. It can be given as a sentence in words, but most often a proportion is an algebraic equation.

The arithmetic equation $\dfrac{3}{5} = \dfrac{21}{35}$ is a proportion because its cross products are equal.

$$3 \times 35 = \mathbf{105} \qquad \text{and} \qquad 5 \times 21 = \mathbf{105}$$

Proportions are solved by using this cross-product rule.

Example #1: $\dfrac{4}{9} = \dfrac{X}{36}$

$4 \times 36 = 9x$

$144 = 9x$

$\dfrac{144}{9} = x$

$16 = x$

Example #2: $\dfrac{72}{1.5} = \dfrac{12}{x}$

$72x = 1.5 \times 12$

$72x = 18$

$x = \dfrac{18}{72}$

$x = .25$ or ¼

Applied Proportion Problems

Many problems can be solved by setting up a **direct proportion** (an increase in one quantity leads to a proportional increase in the other quantity) or by setting up **equivalent rates**.

Example: In one day you earn $75 for 8 hours of work. If you work 37.5 hours for the week, what will your weekly pay be?

$\dfrac{8 \text{ hours}}{37.5 \text{ hours}} = \dfrac{\$75}{x}$

$8x = 75 \times 37.5$

$8x = 2812.5$

$x = \dfrac{2812.5}{8}$

$x = \$351.56$

or

$\dfrac{8 \text{ hours}}{\$75} = \dfrac{37.5 \text{ hours}}{x}$

$8x = 75 \times 37.5$

$8x = 2812.5$

$x = \dfrac{2812.5}{8}$

$x = \$351.56$