

# Properties of Exponents

An exponent (also called power or degree) tells us how many times the base will be multiplied by itself. For example  $x^5$ , the exponent is 5 and the base is  $x$ . This means that the variable  $x$  will be multiplied by itself 5 times. You can also think of this as  $x$  to the fifth power.

Below is a list of properties of exponents:

Properties	General Form	Application	Example
<b>Product Rule</b> <i>Same base add exponents</i>	$a^m a^n$	$a^{m+n}$	$x^5 x^3 = x^{5+3} = x^8$
<b>Quotient Rule</b> <i>Same base subtract exponents</i>	$\frac{a^m}{a^n}$	$a^{m-n}$	$\frac{x^9}{x^5} = x^{9-5} = x^4$
<b>Power Rule I</b> <i>Power raised to a power multiply exponents.</i>	$(a^m)^n$	$a^{mn}$	$(x^3)^4 = x^{3 \cdot 4} = x^{12}$
<b>Power Rule II</b> <i>Product to power distribute to each base</i>	$(ab)^m$	$a^m a^n$	$(4x^3)^2 = 4^2 x^{3 \cdot 2} = 16x^6$
<b>Negative Exponent I</b> <i>Flip and change sign to positive</i>	$a^{-m}$	$\frac{1}{a^m}$	$x^{-3} = \frac{1}{x^3}$
<b>Negative Exponent II</b> <i>Flip and change sign to positive</i>	$\frac{1}{a^{-m}}$	$a^m$	$\frac{1}{x^{-5}} = x^5$
<b>Zero Exponent</b> <i>Anything to the zero power (except 0) is one</i>	$a^0$	$a^0 = 1$	$(-4x)^0 = 1$

- It is important to note that none of these applications can occur if the bases are not the same.

For example,  $\frac{x^5}{y^3}$  cannot be simplified.

## Using a Combination of Rules

At one point, you may be asked to use a combination of these properties.

Example:

- $\frac{(2^3y^2)^5}{2^{10}y^{16}}$  → Power Rule
- $\frac{2^{3 \cdot 5}y^{2 \cdot 5}}{2^{10}y^{16}}$
- $\frac{2^{15}y^{10}}{2^{10}y^{16}}$  → Quotient Rule
- $2^{15-10}y^{10-16}$
- $2^5y^{-6}$  → Negative Exponent
- $\frac{32}{y^6}$

Example:

- $\left(\frac{p^{-4}q}{r^{-3}}\right)^{-3}$  → Power Rule
- $\frac{p^{-4 \cdot -3}q^{1 \cdot -3}}{r^{-3 \cdot -3}}$  **Note:** When a base does not have an exponent there is really a one as the power. So that,  $q$  is understood as  $q^1$
- $\frac{p^{12}q^{-3}}{r^9}$  → Negative Exponents
- $\frac{p^{12}}{q^3r^9}$