Section 4-2: Operations on Functions

Operations on functions are similar to operations on numbers. Each operation has a formal definition which produces a new function!!

Operations on Functions

Each function is defined for all x in the domains of both f and g.

- ***1) Sum of f and g:*** $(f + g)(x) = f(x) + g(x)$
- ***2) Difference of f and g:*** $(f - g)(x) = f(x) - g(x)$
- ***3) Product of f and g:*** $(f \cdot g)(x) = f(x) \cdot g(x)$
- ***4) Quotient of f and g:*** $(f/g)(x) = f(x)/g(x)$, ***g(x) not equal to 0***

Sample Problems

Let $f(x) = x + 3$ and $g(x) = x - 2$

1) *The sum* $= (f + g)(x) = f(x) + g(x) = x + 3 + x - 2 = 2x + 1$

2) *The difference* $= (f - g)(x) = f(x) - g(x) = (x + 3) - (x - 2) = 5$

3) *The product* $= (f \cdot g)(x) = f(x) \cdot g(x) = (x + 3)(x - 2) = x^2 + x - 6$

4) *The quotient* $= (f/g)(x) = f(x)/g(x) = (x + 3)/(x - 2)$, *x does not = 2*

Another way of combining functions is the composite function. The composite function is denoted by:

$$f \circ g$$

and is = f(g(x)). x must be in the domain of g(x) and g(x) must be in the domain of f(x).

<u>Example 1</u>

Find f(g(x)) and g(f(x)) for the functions $f(x) = x + 2$ ***and*** $g(x) = x^2$

$f(g(x)) = f(x^2) = x^2 + 2$. ***The function g(x) squares any number. The function f(x) simply adds two to any number. G(x) is applied first (it squares x) and then f(x) is applied (it adds two). The domain is all real numbers.***

$g(f(x)) = g(x + 2) = (x + 2)^2$ ***This time, f(x) is applied*** *first and then g(x). Take any number and add two first then square the result. The domain is all real numbers. Notice that the answers are not the same illustrating that the composite function is not commutative. This means the order in which the problem is written is important.*

Example two

Find $f(g(x))$ and $g(f(x))$ for the functions $f(x) = 1/x$ and $g(x) = 2x$.

$f(g(x)) = f(2x) = 1/(2x)$. The domain is all real numbers but zero.

$g(f(x)) = g(1/x) = 2/x$. The domain is again all real numbers but zero.

Example three

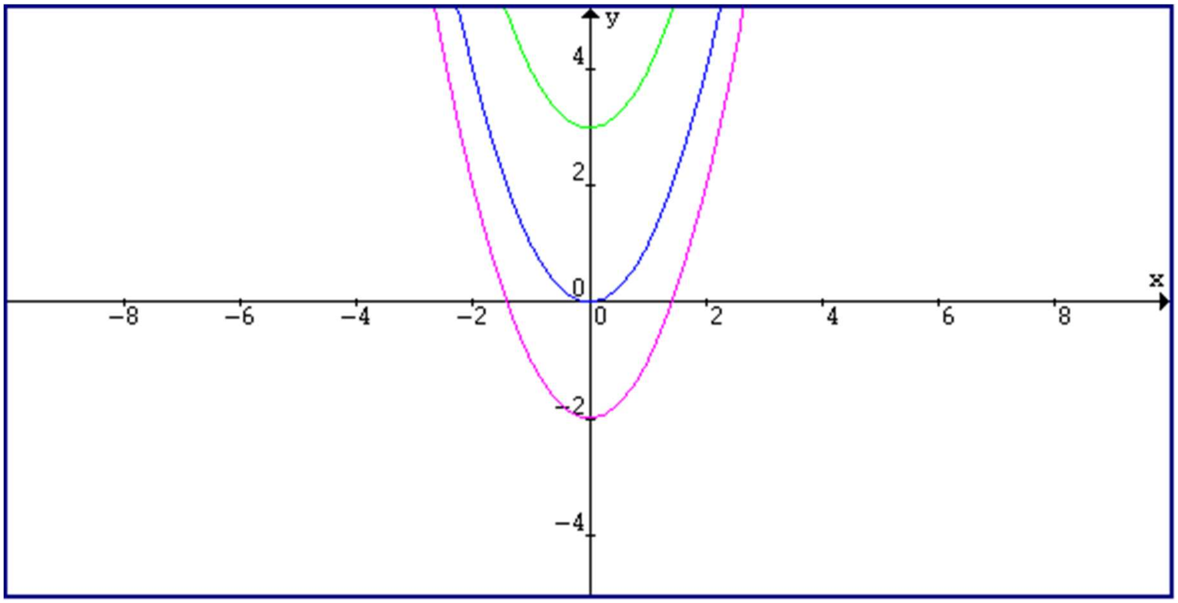
Find $f(g(x))$ and $g(f(x))$ for the functions $f(x) = x + 3$ and $g(x) = 2/x$

$f(g(x)) = f(2/x) = 2/x + 3$. The domain is all real numbers but 0.

$g(f(x)) = g(x + 3) = 2/(x + 3)$. The domain is all real numbers but -3.

Sample Problems

1) On the same axes graph $f(x) = x^2$, $g(x) = x^2 + 3$, and $h(x) = x^2 - 2$



The effect of adding three, simply moved the graph up 3 units while the effect of subtracting two simply moved the graph down two units.

2) Let $f(x) = 4x$, $g(x) = 3x + 2$ and $h(x) = x^2$

Find $f(g(h(3)))$

$$= f(g(9)) = f(29) = 116$$

Find $h(g(f(-1)))$

$$= h(g(-4)) = h(-10) = 100$$

Find $h(h(h(2)))$

$$= h(h(4)) = h(16) = 256$$

Find $h(g(f(x)))$

$$= h(g(4x)) = h(12x + 2) = (12x + 2)^2$$