<u>Notes on Ratios, Rates, and Proportions</u>

<u>ratio</u>–A comparison of two numbers or quantities. They are measured in the <u>same or similar units</u>.

Example: If the ratio of adults to children is 2 to 5, then there are two adults for every 5 children. So, if there are 50 children in attendance, then there are 20 adults.

Ratios can be written in three ways: 2 to 5 2:5 $\frac{2}{5}$

<u>rate</u>–A special ratio that compares two quantities measured in <u>different types of units</u>.

Example: The water dripped at a rate of 2 liters every 3 hours $\rightarrow \frac{2\text{ L}}{3\text{ hours}}$

<u>unit rate</u>–a rate with a denominator of 1.

Example: Shelby drove 70 mph. $\rightarrow \frac{70\text{ miles}}{1\text{ hour}}$

<u>proportion</u>–An equation of two equivalent ratios.

Example: a 10 pound bag of M&Ms costs \$8. How much does each pound of M&Ms cost?

$$\frac{\$8}{10\text{ pounds}} = \frac{\$x}{1\text{ pound}}$$

$x = \$0.80$

The M&Ms cost \$0.80 per pound.

<u>equivalent proportions</u>–proportions that are essentially the same although they look a little different.

How can you tell if proportions are equivalent? The values that are diagonal are the same.

Example: $\frac{\$37}{100\%} = \frac{x}{70\%}$ is equivalent to $\frac{\$37}{x} = \frac{100\%}{70\%}$ and $\frac{x}{\$37} = \frac{70\%}{100\%}$

but they are **NOT** equivalent to $\frac{\$37}{x} = \frac{70\%}{100\%}$

Note: the equivalent proportions all have \$37 diagonal to 70% and x diagonal to 100%. The proportion that is not equivalent does not have this quality.

Solving proportions

You can solve a proportion many ways. First remove the units.

Example 1 $\frac{\$37}{100\%}=\frac{x}{70\%} \rightarrow \frac{37}{100}=\frac{x}{70}$

Now solve algebraically.

$$70\cdot\frac{37}{100}=\frac{x}{70}\cdot 70$$

$$^{7}\cancel{70}\cdot\frac{37}{^{10}\cancel{100}}=\frac{x}{\cancel{70}_{1}}\cdot\cancel{70}^{1}$$

$$\frac{259}{10}=x$$

$$25.9=x$$

$$x=\$25.90$$

Note: A shortcut here is to multiply the two diagonal values that are known and divide them by the value diagonal to the variable (unknown).

$$x=\frac{37\cdot 70}{100}=25.9 \text{ or } \$25.90$$

Example 2 $\frac{\$50}{3\ hours}=\frac{\$250}{x\ hours} \rightarrow \frac{50}{3}=\frac{250}{x}$

Again, start by removing the units and solving algebraically.

$$x\cdot\frac{50}{3}=\frac{250}{x}\cdot x$$

$$x\cdot\frac{50}{3}=\frac{250}{\cancel{x}_{1}}\cdot\cancel{x}_{1}$$

$$\frac{50x}{3}=250$$

$$\frac{3}{50}\cdot\frac{50x}{3}=250\cdot\frac{3}{50}$$

$$\frac{^{1}\cancel{3}}{_{1}\cancel{50}}\cdot\frac{\cancel{50}^{1}x}{\cancel{3}_{1}}={}^{5}\cancel{250}\cdot\frac{3}{\cancel{50}_{1}}$$

$$x=15$$

Note: A shortcut here is to use the Giant One and write equivalent ratios.

$$\frac{50^{\cdot 5}}{3_{\cdot 5}}=\frac{250}{x}\rightarrow x=15$$

Note: The same can be done vertically. Imagine the equivalent proportion:

$$\frac{50}{250}=\frac{3}{x}$$

We can see that if we multiply the numerator by 5, we get the denominator. So, we do this on both sides of the proportion.

Example 3 $\dfrac{3x+2 \text{ miles}}{14 \text{ hours}} = \dfrac{x-5 \text{ miles}}{9 \text{ hours}} \rightarrow \dfrac{3x+2}{14} = \dfrac{x-5}{9}$

Again, start by removing the units and then solve algebraically.

$$9 \cdot \frac{3x+2}{14} = \frac{x-5}{9} \cdot 9$$

$$\frac{9(3x+2)}{14} = \frac{x-5}{\cancel{9}_1} \cdot \cancel{9}_1$$

$$\frac{27x+18}{14} = x-5$$

Now multiply both sides of the equation by 14.

$$14 \cdot \frac{27x+18}{14} = (x-5) \cdot 14$$

$$27x+18 = 14x-70$$
$$-14x \quad -14x$$
$$13x+18 = -70$$
$$-18 \quad -18$$
$$13x = -88$$
$$\frac{13x}{13} = \frac{-88}{13}$$
$$x = -6\frac{10}{13}$$

Note: A shortcut here is to multiply the values on the two diagonals. From there, solve like usual.

$$\frac{3x+2}{14} = \frac{x-5}{9}$$

$$9(3x+2) = 14(x-5)$$

$$27x+18 = 14x-70$$

$$x = -6\frac{10}{13}$$

Note on shortcuts: The shortcut in Example 3 is a commonly used shortcut. It is often referred to as "cross multiplying".

Graphing proportions

We can graph our information on a coordinate graph. One unit is on the x-axis and the other is on the y-axis.

Examples:

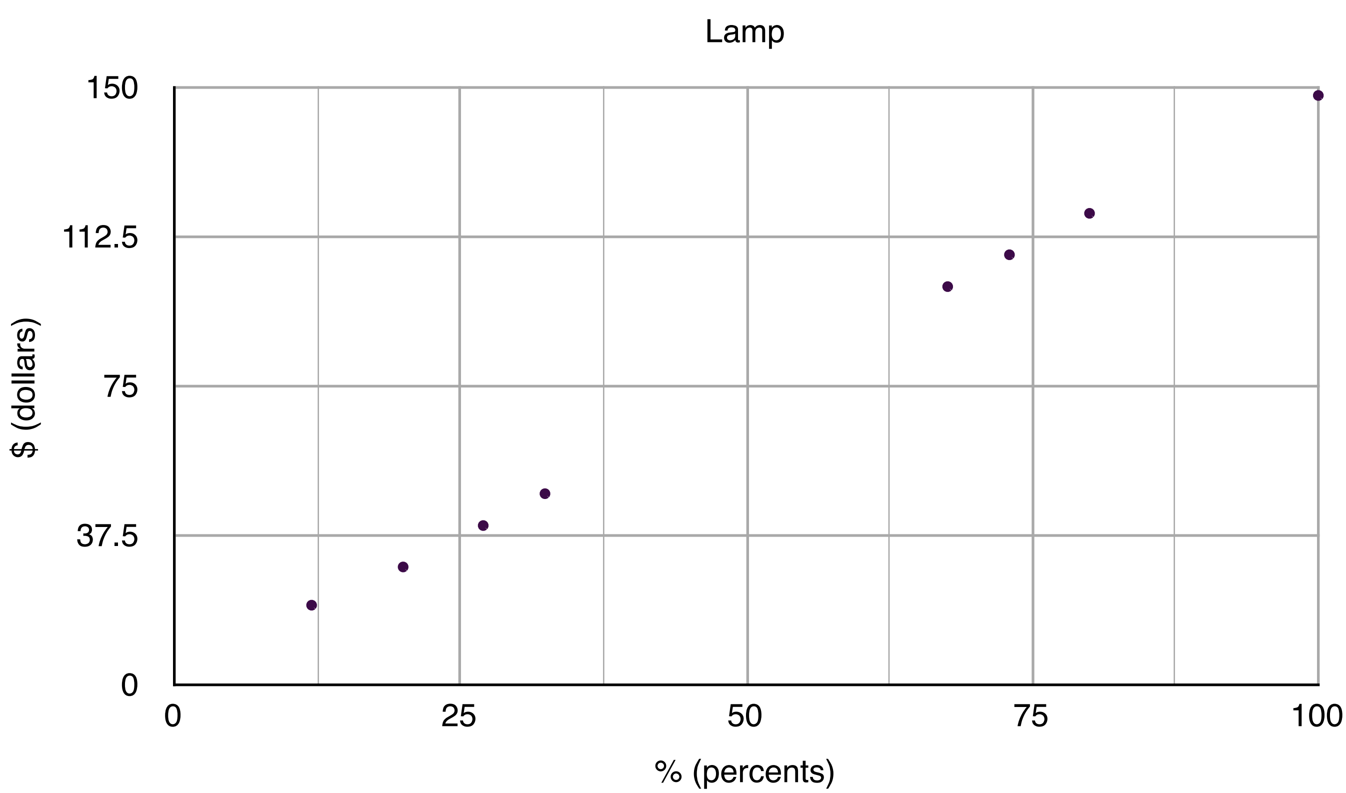
A lamp is originally $148.

(a) It is on sale for 20% off. What is the discount?
(b) It is on sale for 20% off. What is the new cost?
(c) It is now $100; what percent are you paying now?
(d) It is now $100; what percent do you save?
(e) You have a coupon for $40 off. What percent do you save?
(f) You have a coupon for $40 off. What percent are you paying now?

Let's put this information in a table, SOLVE USING PROPORTIONS, and then graph it.

% (percents)	100	20	80					**x**
$ (dollars)	148			100	48	40	108	**y**

% (percents)	100	20	80	$67.\overline{567}$	$32.\overline{432}$	$27.\overline{027}$	$72.\overline{972}$	**x**
$ (dollars)	148	**29.60**	**118.40**	100	48	40	108	**y**



Proportional relationships, when graphed, are **<u>linear</u> and pass through the <u>origin</u>, (0,0).**