Algebraic Substitution

One method of solving system of equations is by algebraic substitution.

Example: Solve $x = 7 - 3y$ and $2x - 4y = -6$.

$2(7 - 3y) - 4y = -6$ Substitute $7 - 3y$ for x in the second equation.

$14 - 6y - 4y = -6$ Solve for y

$-10y = -20$

$y = 2$

Substitute 2 for y in either one of the two original equations to find the value of x.

$x + 3(2) = 7$

$x + 6 = 7$

$x = 1$ The solution of this system is $(1, 2)$

Elimination Using Addition & Subtraction

In systems of equations where the coefficient of the x or y terms are additive inverses, solve the system by adding the equations. Because one of the variables is eliminated, this method is called **elimination**.

Example: Use elimination to solve the system of equations.

$x - 3y = 7$ and $3x + 3y = 9$

Add the two equations

$$\begin{array}{r} x - 3y = 7 \\ \underline{3x + 3y = 9} \\ 4x = 16 \\ x = 4 \end{array}$$

Substitute 4 for x in either original equation and solve for y.

$$\begin{array}{r} 4 - 3y = 7 \\ -3y = 7 - 4 \\ -3y = 3 \\ y = -1 \end{array}$$

The solution of the system is $(4, -1)$

Elimination Using Multiplication

Some systems of equations cannot be solved simply by adding or subtracting the equations. One or both equations must first be multiplied by a number before the system can be solved by elimination. Consider the following example:

Example: Use elimination to solve the following system of equations.

$$x + 10y = 3 \quad \text{and} \quad 4x + 5y = 5$$

$x + 10y = 3$ multiply $x + 10y = 3$ by -4. $-4x - 40y = -12$

$4x + 5y = 5$ Then add the equations. $\underline{4x + 5y = 5}$

$$-35y = -7$$

$$y = \frac{1}{5}$$

Substitute $\frac{1}{5}$ for y into either original equation and solve for x.

$$x + 10\left(\frac{1}{5}\right) = 3$$

$$x + 2 = 3$$

$$x = 1$$

The solution of the system is $\left(1, \frac{1}{5}\right)$

Graphing Systems of Equations

Two or more linear equations involving the same variables form a **system of equations**. The solution set for the system is the set of ordered pairs that satisfy both equations. One method for solving a system of equations is to graph the equations on the same coordinate plane.

Example: Solve each system of equations by graphing.

$x + y = 2$

$x - y = 4$

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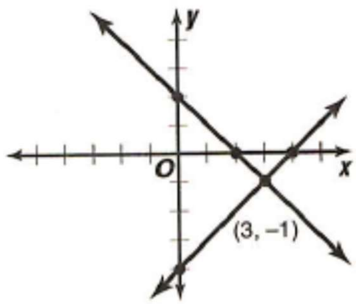
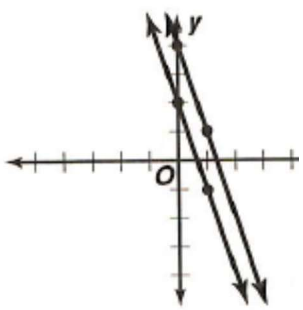


The point $(3, -1)$ lies on both lines, thus $(3, -1)$ is the solution set for the system of equations

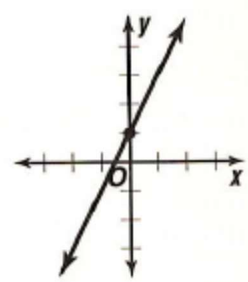
$3x + y = 2$ no solution

$3x + y = 4$



$y = 2x + 1$ infinitely many solutions

$2y = 4x + 2$



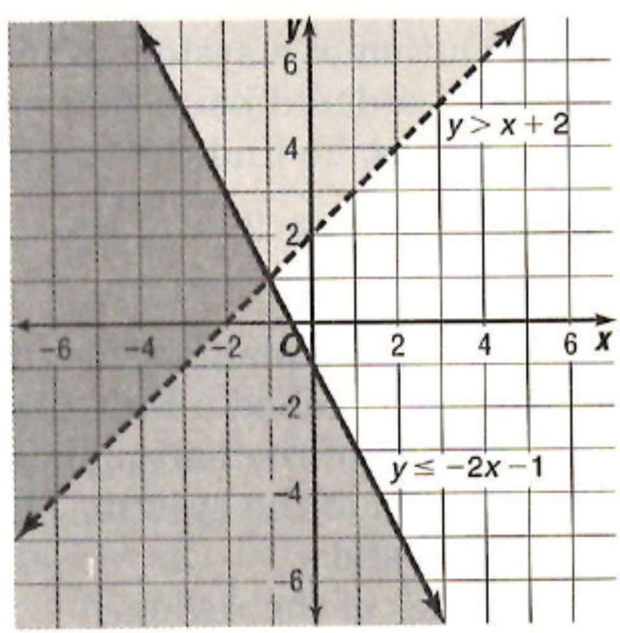
Graphing Systems of Inequalities

The solution of a system of inequalities is the set of all ordered pairs that satisfy both inequalities. To find the solution of the system

$$y > x + 2$$

$$y \leq -2x - 1$$

graph each inequality. The graph of each inequality is called a **half-plane**. The intersection of the half-planes represents the solution of the system. The graphs of $y = x + 2$ and $y = -2x - 1$ are the boundaries of the region.



An inequality containing an absolute value expression can be graphed by graphing an equivalent system of two inequalities.