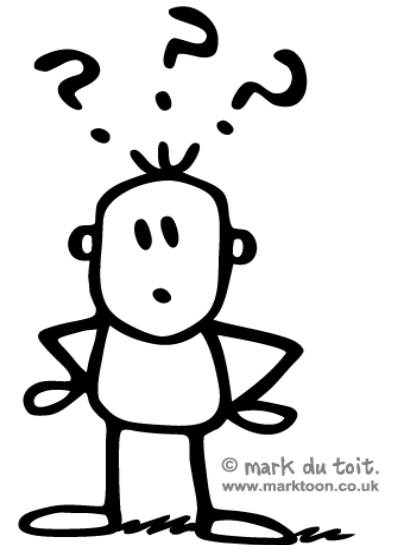


Dilations

Lesson 6.4

Video:

What is a Dilation?



Dilations in the Coordinate Plane

A dilation with a scale factor of k will be:

- an enlargement if $k > 1$
- a reduction if $0 < k < 1$
- the same as the original figure if $k = 1$

When the center of dilation in the coordinate plane is the origin, each coordinate of the preimage is multiplied by the scale factor k to find the coordinates of the image.

$$(x, y) \rightarrow (kx, ky)$$

Enlargement vs Reduction:

State whether each scale factor k would result in an enlargement or reduction of the original image.

1. $k = 5$ enlargement
2. $k = 0.25$ reduction
3. $k = 1/2$ reduction
4. $k = 3$ enlargement

Example

- 1. A triangle has vertices $A(0, 0)$, $B(8, 0)$, and $C(3, -2)$. Find the coordinates of the triangle after a dilation with a scale factor of 4.**

The dilation is $(x, y) \rightarrow (4x, 4y)$. Multiply the coordinates of each vertex by 4.

$$A(0, 0) \rightarrow (4 \cdot 0, 4 \cdot 0) \rightarrow (0, 0)$$

$$B(8, 0) \rightarrow (4 \cdot 8, 4 \cdot 0) \rightarrow (32, 0)$$

$$C(3, -2) \rightarrow [4 \cdot 3, 4 \cdot (-2)] \rightarrow (12, -8)$$

So, the coordinates after the dilation are $A'(0, 0)$, $B'(32, 0)$, and $C'(12, -8)$.

Example

A triangle has vertices $D(1,2)$, $E(0,4)$, and $F(1, -1)$. Find the coordinates of the triangle after a dilation with a scale factor of 3.

$$D(1,2) \rightarrow (1 \cdot 3, 2 \cdot 3) \rightarrow D'(3,6)$$

$$E(0,4) \rightarrow (0 \cdot 3, 4 \cdot 3) \rightarrow E'(0,12)$$

$$F(1,-1) \rightarrow (1 \cdot 3, -1 \cdot 3) \rightarrow F'(3,-3)$$

Got It? Do this problem to find out.

- a. A figure has vertices $W(-2, 4)$, $X(1, 4)$, $Y(-3, -1)$, and $Z(3, -1)$. Find the coordinates of the figure after a dilation with a scale factor of 2.

$$W(-2, 4) \rightarrow (-2 \cdot 2, 4 \cdot 2) \rightarrow W(-4, 8)$$

$$X(1, 4) \rightarrow (1 \cdot 2, 4 \cdot 2) \rightarrow X(2, 8)$$

$$Y(-3, -1) \rightarrow (-3 \cdot 2, -1 \cdot 2) \rightarrow Y(-6, -2)$$

$$Z(3, -1) \rightarrow (3 \cdot 2, -1 \cdot 2) \rightarrow Z(6, -2)$$

Got It? Do this problem to find out.

A figure has vertices $A(1,3)$, $B(4,3)$, $C(5, 1)$, and $D(2,1)$. Find the coordinates of the triangle after a dilation with a scale factor of $\frac{1}{2}$.

$$A(1,3) \rightarrow \left(\frac{1}{2} \cdot 1, \frac{1}{2} \cdot 3\right) \rightarrow A'\left(\frac{1}{2}, 1\frac{1}{2}\right)$$

$$B(4,3) \rightarrow \left(\frac{1}{2} \cdot 4, \frac{1}{2} \cdot 3\right) \rightarrow B'\left(2, 1\frac{1}{2}\right)$$

$$C(5,1) \rightarrow \left(\frac{1}{2} \cdot 5, \frac{1}{2} \cdot 1\right) \rightarrow C'\left(2\frac{1}{2}, \frac{1}{2}\right)$$

$$D(2,1) \rightarrow \left(\frac{1}{2} \cdot 2, \frac{1}{2} \cdot 1\right) \rightarrow D'\left(1, \frac{1}{2}\right)$$

Got It? Do this problem to find out.

A triangle has vertices $L(0,4)$, $M(8,4)$, and $N(8, -1)$. Find the coordinates of the triangle after a dilation with a scale factor of $\frac{3}{4}$.

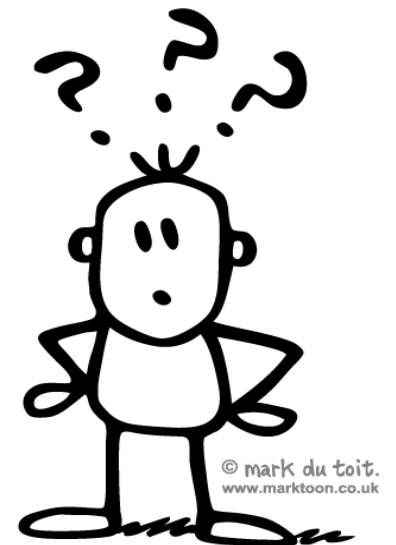
$$L(0,4) \rightarrow \left(0 \cdot \frac{3}{4}, 4 \cdot \frac{3}{4}\right) \rightarrow L'(0,3)$$

$$M(8,4) \rightarrow \left(8 \cdot \frac{3}{4}, 4 \cdot \frac{3}{4}\right) \rightarrow M'(6,3)$$

$$N(8,-1) \rightarrow \left(8 \cdot \frac{3}{4}, -1 \cdot \frac{3}{4}\right) \rightarrow N'\left(6, -\frac{3}{4}\right)$$

Video:

How do you use a scale factor to graph a dilation?



Example

- 2.** A figure has vertices $J(3, 8)$, $K(10, 6)$, and $L(8, 2)$. Graph the figure and the image of the figure after a dilation with a scale factor of $\frac{1}{2}$.

Step 1: Find the coordinates after the dilation

$$J(3, 8) \rightarrow \left(\frac{1}{2} \cdot 3, \frac{1}{2} \cdot 8\right) \rightarrow J'\left(\frac{3}{2}, 4\right)$$

$$K(10, 6) \rightarrow \left(\frac{1}{2} \cdot 10, \frac{1}{2} \cdot 6\right) \rightarrow K'(5, 3)$$

$$L(8, 2) \rightarrow \left(\frac{1}{2} \cdot 8, \frac{1}{2} \cdot 2\right) \rightarrow L'(4, 1)$$

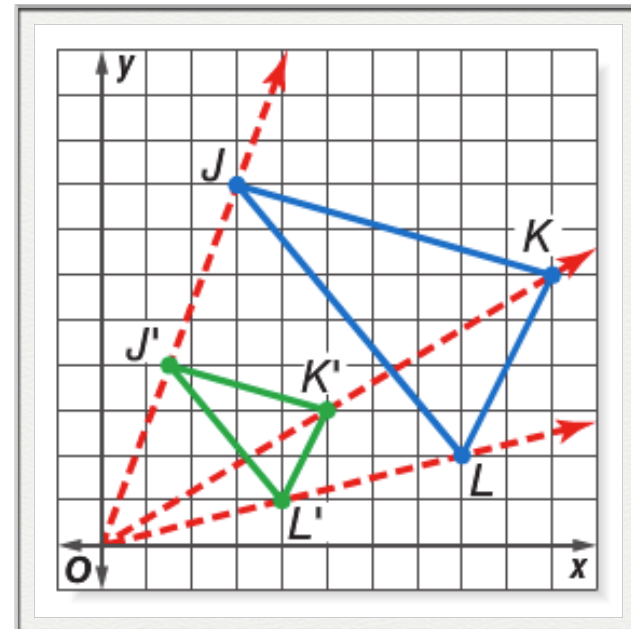
Example

Step 2: Graph both sets of coordinates

$$J(3, 8) \rightarrow J'\left(\frac{3}{2}, 4\right)$$

$$K(10, 6) \rightarrow K'(5, 3)$$

$$L(8, 2) \rightarrow L'(4, 1)$$



Check Draw lines through the origin and each of the vertices of the original figure. The vertices of the dilation should lie on those same lines. ✓

Example

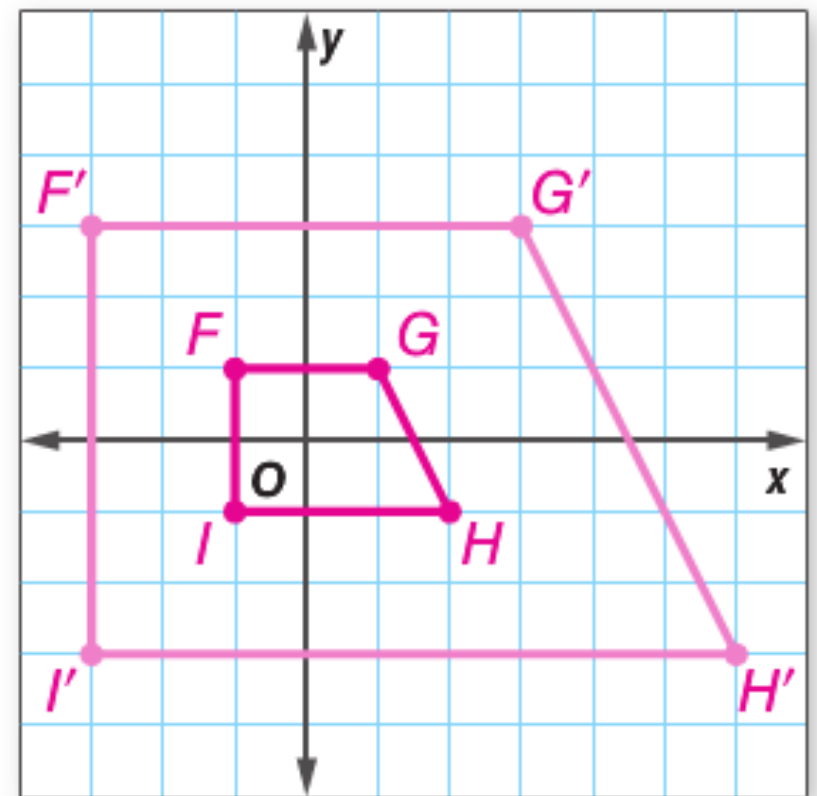
A figure has vertices $F(-1, 1)$, $G(1, 1)$, $H(2, -1)$, and $I(-1, -1)$. Graph the figure and the image of the figure after a dilation with a scale factor of 3.

$$F(-1, 1) \rightarrow F'(-3, 3)$$

$$G(1, 1) \rightarrow G'(3, 3)$$

$$H(2, -1) \rightarrow H'(6, -3)$$

$$I(-1, -1) \rightarrow I'(-3, -3)$$

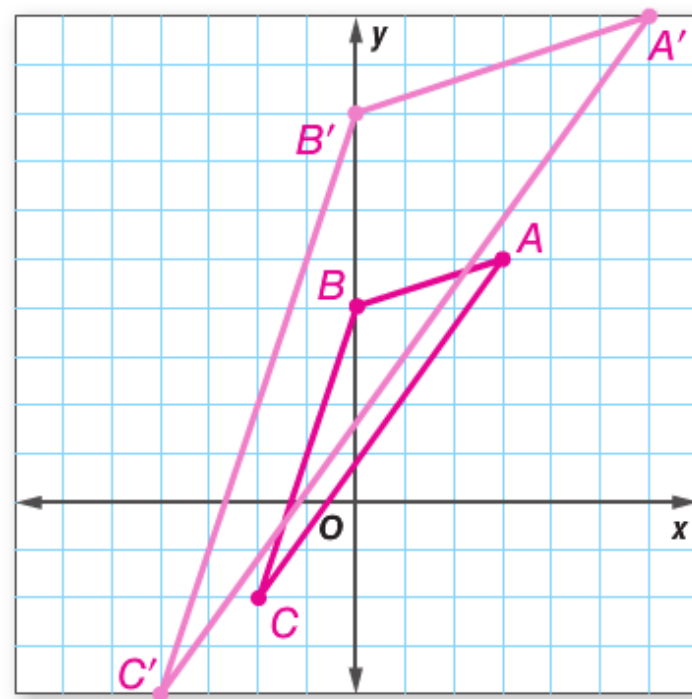


Got It? Do this problem to find out.

Find the coordinates of the vertices of each figure after a dilation with the given scale factor k . Then graph the original image and the dilation.

$$A(3, 5), B(0, 4), C(-2, -2); k = 2$$

$$A'(6, 10), B'(0, 8), C'(-4, -4)$$

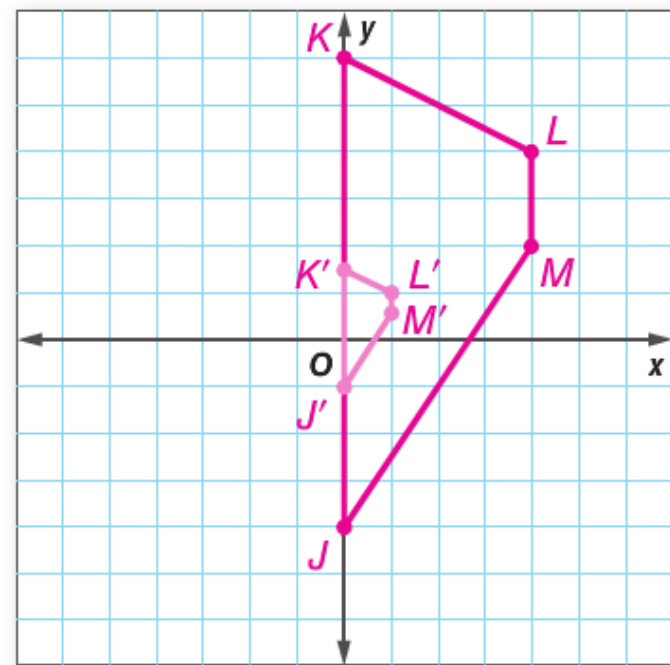


Got It? Do this problem to find out.

Find the coordinates of the vertices of each figure after a dilation with the given scale factor k . Then graph the original image and the dilation.

$$J(0, -4), K(0, 6), L(4, 4), M(4, 2); k = \frac{1}{4}$$

$$\underline{J'(0, -1), K'(0, 1\frac{1}{2}), L'(1, 1), M'(1, \frac{1}{2})}$$



Example

- 3.** Through a microscope, the image of a grain of sand with a 0.25-millimeter diameter appears to have a diameter of 11.25 millimeters. What is the scale factor of the dilation?

Write a ratio comparing the diameters of the two images.

$$\frac{\text{diameter in dilation}}{\text{diameter in original}} = \frac{11.25}{0.25} = 45$$

So, the scale factor of the dilation is 45.

Example

Lucas wants to enlarge a 3- by 5-inch photo to a $7\frac{1}{2}$ - by $12\frac{1}{2}$ -inch photo. What is the scale factor of the dilation?

* Choose only one side to compare in the ratio *

$$\frac{\textit{dilation}}{\textit{original}} = \frac{7\frac{1}{2}}{3} = \frac{15}{2} \cdot \frac{1}{3} = \frac{5}{2} = \boxed{2\frac{1}{2} \textit{ or } 2.5}$$

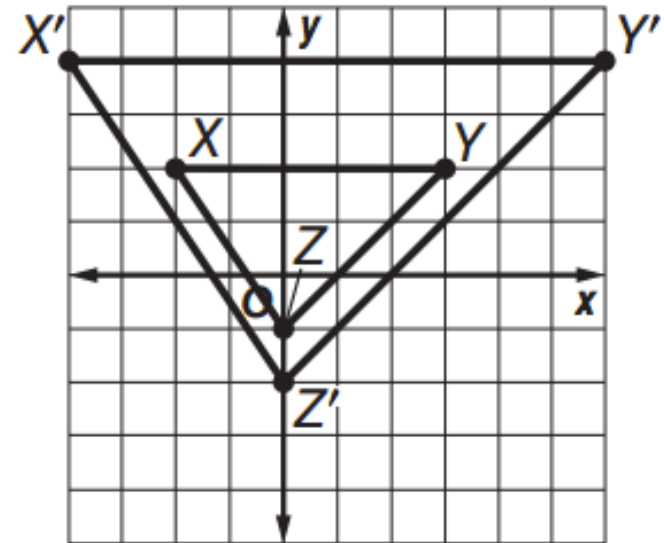
Example

In the figure at the right, $\triangle X'Y'Z'$ is a dilation of $\triangle XYZ$. Find the scale factor of the dilation, and classify it as an enlargement or a reduction.

* Choose only one point to compare in the ratio *

$$Z(0, -1) \rightarrow Z'(0, -2)$$

$$\frac{\text{dilation}}{\text{original}} = \frac{-2}{-1} = \boxed{2}$$



Got It? Do this problem to find out.

The pupil of Josh's eye is 6 mm in diameter. His doctor uses medicine to dilate his pupils so that they are 9 mm in diameter. What is the scale factor of the dilation?

Write a ratio comparing the diameters of the two images.

$$\frac{\text{diameter in dilation}}{\text{diameter in original}} = \frac{9}{6} = \frac{3}{2} = \boxed{1\frac{1}{2} \text{ or } 1.5}$$

Got It? Do this problem to find out.

STEM Mrs. Bowen's homeroom is creating a Web page for their school's Intranet site. They need to reduce a scanned photograph so it is 720 pixels by 320 pixels. If the scanned photograph is 1,080 pixels by 480 pixels, what is the scale factor of the dilation? (Example 3) _____

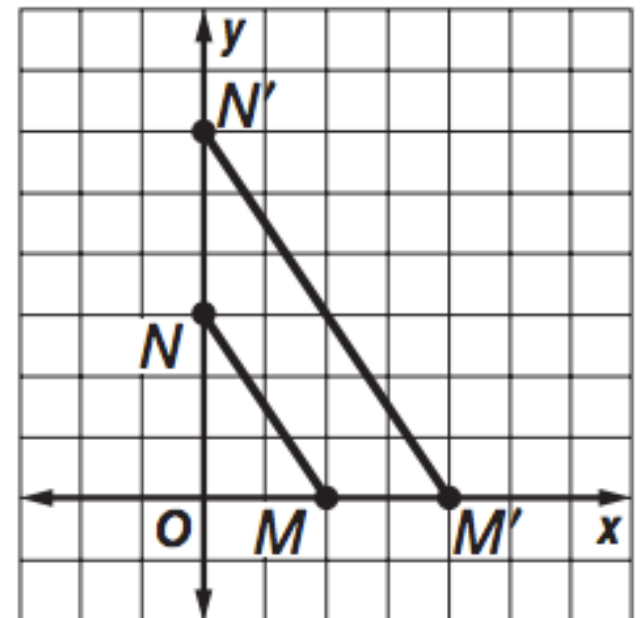
$$\frac{\textit{dilation}}{\textit{original}} = \frac{720}{1080} = \frac{2}{3}$$

Got It? Do this problem to find out.

The graph shows segment $M'N'$ is a dilation of segment MN .
What is the scale factor of the dilation?

$$N(0,3) \rightarrow N'(0,6)$$

$$\frac{\text{dilation}}{\text{original}} = \frac{6}{3} = \boxed{2}$$



Homework:

Pg. 491-494

#1-9 and #14-21

(all)