

5.4 Solving Equations with Infinite or No Solutions

So far we have looked at equations where there is exactly one solution. It is possible to have more than one solution in other types of equations that are not linear, but it is also possible to have no solutions or infinite solutions. No solution would mean that there is no answer to the equation. It is impossible for the equation to be true no matter what value we assign to the variable. Infinite solutions would mean that any value for the variable would make the equation true.

No Solution Equations

Let's look at the following equation:

$$2x + 3 = 2x + 7$$

Note that we have variables on both sides of the equation. So we'll subtract $2x$ from both sides to eliminate the $2x$ on the right side of the equation. However, something odd happens.

$$\begin{array}{r} 2x + 3 = 2x + 7 \\ -2x \quad -2x \\ \hline 3 = 7 \end{array}$$

That can't be right! We know that three doesn't equal seven. It is a false statement to say $3 = 7$, so we know that there can be no solution. Does that make sense though? Well if we took twice a number and added three, would it ever be the same as twice a number and adding seven?

Let's look at another example equation:

$$3(x + 4) = 3x + 11$$

Note that we need to simplify and that there are variables on both sides of the equation. So we'll first multiply through the parentheses with the distributive property and then subtract $3x$ from both sides to eliminate the $3x$ on the right side of the equation.

$$\begin{array}{r} 3(x + 4) = 3x + 11 \\ 3x + 12 = 3x + 11 \\ -3x \quad -3x \\ \hline 12 = 11 \end{array}$$

We again get a false statement and therefore we know there are no solutions. Sometimes we use the symbol \emptyset to represent no solutions. That symbol means "empty set" which means that the set of all answers is empty. In other words, there is no answer. So if we want to use \emptyset to represent no solution, we may.

Infinite Solutions Equations

Let's look at the following equation:

$$2x + 3 = 2x + 3$$

Note that we have variables on both sides of the equation. So we'll subtract $2x$ from both sides to eliminate the $2x$ on the right side of the equation. However, something different happens this time.

$$\begin{array}{r} 2x + 3 = 2x + 3 \\ -2x \quad -2x \\ \hline 3 = 3 \end{array}$$

When does three equal three? All the time! This means that it doesn't matter what value we substitute for x , the equation will always be true. Go ahead and try plugging in a couple of your favorite numbers to verify this is true.

Also note that twice a number plus three is equal to itself in our original equation. When is something equal to itself? Always! So there are infinite solutions. Sometimes we use the symbol ∞ , which means infinity, to represent infinite solutions.

Let's look at one more example with simplification necessary.

$$\begin{array}{r} -2(x + 3) = -2x - 6 \\ -2x - 6 = -2x - 6 \\ +2x \quad +2x \\ \hline -6 = -6 \end{array}$$

We again get a statement that is always true and therefore we know there are infinite solutions.

Creating Multi-Step One Solution Equations

Now that we understand how to solve the different types of equations, we should be able to create them. To create a one solution equation, we can honestly create an equation using any number we want as long as we don't have the same amount of variables on both sides of the equation. For example, this equation would have a single solution because the variables will not "disappear" from both sides of the equation as we simplify:

$$x + 2x + 3 + 4 = 5x + 6x + 7 + 8$$

What is the solution for that equation?

$$\begin{array}{r} 3x + 7 = 11x + 15 \\ -3x \quad -3x \\ \hline 7 = 8x + 15 \\ -15 \quad -15 \\ \hline -8 = 8x \\ \div 8 \quad \div 8 \\ \hline -1 = x \end{array}$$

Creating Multi-Step No Solution Equations

To create a no solution equation, we can need to create a mathematical statement that is always false. To do this, we need the variables on both sides of the equation to cancel each other out and have the remaining values to not be equal. Take this simple equation as an example.

$$\begin{array}{r} x + 1 = x + 2 \\ -x \quad -x \\ \hline 1 = 2 \end{array}$$

Since one does not equal two, we know we have an equation with no solution. However, we want multi-step equations, so we'll need to make it a bit more complex. Let's look at this example:

$$x + 2x + 1 = 3x + 2 + 3$$

$$\begin{array}{r} 3x + 1 = 3x + 5 \\ -3x \quad -3x \\ \hline 1 = 5 \end{array}$$

Notice that we combined like terms first and then eliminated the variable from one side. When that happened, the variable on the other side was eliminated as well, giving us a false result. There the key to creating equations with no solutions is to have the coefficients (number in front of the variable) match and the constants (regular numbers after that) not match.

Creating Multi-Step Infinite Solutions Equations

If we needed to create a false math statement for no solutions, what type of math statement do we need to create one with infinite solutions? Yes, we need one that will be always true. Consider the following example:

$$x + 2x + 3 + 3 = 3(x + 2)$$

$$\begin{array}{r} 3x + 6 = 3x + 6 \\ -3x \quad -3x \\ \hline 6 = 6 \end{array}$$

Again, the coefficients matched after we combined like terms and used the distributive property, but in this case the constants also matched. This gives us the true statement that six does equal six. Therefore there are infinite solutions. Let's look at one more example.

$$4(x + 1) = 4x + 4$$

$$4x + 4 = 4x + 4$$

We should be able to stop here as we notice that the two sides are exactly the same. Four times a number plus four is always equal to four times that number plus four. Therefore there are infinite solutions.

Lesson 5.4

Solve the following equations. Some equations will have a single answer, others will have no solution, and still others will have infinite solutions.

1. $2x + 2x + 2 = 4x + 2$

2. $3(x - 1) = 2x + 9$

3. $2x + 8 = 2(x + 4)$

4. $2x - x + 7 = x + 3 + 4$

5. $-2(x + 1) = -2x + 5$

6. $4x + 2x + 2 = 3x - 7$

7. $2(x + 2) + 3x = 2(x + 1) + 1$

8. $4(x - 1) = \frac{1}{2}(x - 8)$

9. $x + 2x + 7 = 3x - 7$

10. $3x - x + 4 = 4(2x - 1)$

11. $4(2x + 1) = 5x + 3x + 9$

12. $10 + x = 5\left(\frac{1}{5}x + 2\right)$

13. $8(x + 2) = 2x + 16$

14. $3 + \frac{3}{2}x + 4 = 4x - \frac{5}{2}x$

15. $\frac{3}{2}(2x + 6) = 3x + 9$

16. $\frac{1}{2}(2 - 4x) + 2x = 13$

17. $12 + 2x - x = 9x + 6$

18. $4x + 1 = 2(2x + 3)$

19. $4(x + 3) - 4 = 8\left(\frac{1}{2}x + 1\right)$

20. $x + 5x + 4 = 3(2x - 1)$

21. $5(x + 2) - 3x = 2(x + 5)$

22. $3x + 1 = 3(x - 1) + 4$

23. $4x + 2x - 5 = 7x - 1$

24. $-2(x + 1) = 2(x - 1)$

25. $2(x + 5) = 2x + 5$

26. $2(3x + 3) = 3(2x + 2)$

27. $2x + 1 - 4 = -2x - 3$

28. $4(x + 1) = 4(2 - x)$

29. $3x + 7x + 1 = 2(5x + 1)$

30. $6(x + 1) + 5 = 13 - 2 + 6x$

Create multi-step equations with the given number of solutions.

31. A single solution

32. Infinite solutions

33. No solution

34. Infinite solutions

35. No solution

36. A single solution

37. No solution

38. A single solution

39. Infinite solutions

40. A single solution

41. Infinite solutions

42. No solution